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THE BABYLONIAN FIRST VISIBILITY OF THE LUNAR CRESCENT: DATA AND CRITERION

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1. *Introduction*

The problem of predicting the first visibility of the lunar crescent attracted attention throughout much of the historical period, from the many nations who used lunar calendars to regulate their activities. The oldest available records that reveal organized interest in this matter date back almost three thousand years — to the time of the Babylonians. Predicting the first visibility of the lunar crescent aroused great interest among medieval Muslim astronomers, largely because the dates of religious practices in Islam — such as the beginning and end of the fasting month of Ramadhan — are determined by a lunar calendar.

In modern times, scientific interest in understanding the visibility of the lunar crescent has been motivated mainly by two factors: (i) the need of historians correctly to interpret past records of nations that used the lunar calendar; and (ii) the need of present-day Muslims to ascertain each month when the lunar crescent may be visible for the first time after conjunction with the sun — and hence to know when to look for it, and also to know when it cannot be seen.

Predicting the earliest visibility of the lunar crescent after conjunction is a matter of considerable complexity. It is a problem where astronomical, atmospheric, optical and human factors are all at work. The fact that even modern astronomers cannot agree on the best criterion for determining the first visibility of the lunar crescent only attests to the complex nature of this matter.

Throughout history, attempts have been made to put forward criteria for predicting when the young crescent will first be seen in any given month. Each attempt has followed either an empirical or a theoretical approach. The empirical approach, which is more frequently employed, is based on analysing a collection of observational data and then formulating a criterion that best fits the observations. On the other hand, the theoretical method is embodied in attempts to resolve the problem through considering the various factors affecting crescent visibility and designing a descriptive mathematical model. While the Babylonian criterion was empirical, the Arab astronomers took mostly a theoretical approach. Recent studies on the subject have presented prediction models from both aspects: empirical and theoretical.

In this paper, we address the observational aspect of the Babylonian approach to the problem of first visibility of the lunar crescent and also consider the criterion that they have possibly used for predicting the first visibility of the crescent.

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2. *The Role of Observations in the Study of First Visibility of the Lunar Crescent*

Real observations of first visibility of the lunar crescent are crucial for the formulation of an empirical model, yet they are equally important for testing any theoretical solution. Whether empirical or theoretical, the reliability of any criterion can be established with confidence only by testing it against real observations of first visibility of the crescent. This critical role of observational data has urged researchers into the problem of predicting the first visibility of the crescent, to compile such data from the astronomical literature.

It was Fotheringham[1] who made the first such collection in 1910, when he compiled 76 observations of the new moon made by August Mommsen and Julius Schmidt at Athens in the second half of the previous century. Fotheringham used these observations to design his criterion for predicting the first visibility of the lunar crescent. The most comprehensive lists of observations made by experienced observers, including those compiled by Fotheringham, have recently been published by Schaefer[2] and by Doggett and Schaefer.[3] These authors compiled and carefully checked observations from a large number of publications as well as from moonwatches that they organized.[4]

The dates of the observational data compiled by Schaefer and Doggett range from 1859 to 1996, and are from various northern and southern latitudes. The total number of the observations they cite is 294, of which 23 are observations of last visibility of the old moon — rather than first visibility of the new moon. One very important aspect of the 271 evening observations is that they are not all positive sightings: 81 are negative observations, i.e., unsuccessful attempts to spot the new moon. Such negative observations are of exceptional importance in determining the limits of first visibility of the lunar crescent.

In this paper, we present the oldest observations of the lunar crescent that have so far come to light. We have extracted 209 positive observations from the Babylonian "Astronomical Diaries", with their dates ranging from –567 to –73 (568 to 74 B.C.). In the following sections we explain the Babylonian source of data, the conversion of the Babylonian dates into Julian dates, and how we determined the exact Julian date of first visibility of the lunar crescent. Finally, we discuss the possible visibility criterion that the Babylonians may have used.

3. *The Babylonian "Astronomical Diaries"*

The ancient Babylonians developed great interest in astronomical observations. This interest was motivated mainly by their concern with astrology, though calendrical needs contributed as well. In fact, there was never any distinction between the astronomers who made observations and the astrologers who interpreted the observations; both tasks were performed by the same people.[5]

From the eighth century B.C. onward, the Babylonians systematically and continuously recorded their astronomical observations on clay tablets. The Babylonian heritage of astronomical cuneiform texts is usually classified, after Sachs,[6] into four categories: (i) "Almanacs", which are yearly lists of various predicted lunar and planetary phenomena, solstices and equinoxes, etc.; (ii) "Goal-Year Texts", which were designed for the prediction of lunar and planetary phenomena based on certain fundamental periods and were prepared from the "Astronomical Diaries" (see below); (iii) "Normal-Star Almanacs", texts on the positions of thirty-one stars, close to the ecliptic, which the Babylonians used for reference and which were denoted "*Normalsterne*" ("Normal stars") by Epping[7] (a list of these stars, with longitude and latitude at the epoch 164 B.C., is given by Stephenson and Walker[8]); and (iv) the "Astronomical Diaries", the only category of interest for the purpose of this study.

The Astronomical Diaries, or more briefly "diaries", is the modern term used to refer to the tablets known in Akkadian as *nasaru ša ginê*, which means "regular watching". These diaries represent records of daily astronomical observations made in the Neo-Babylonian period by *professionals* who, according to excavated late documents, were employed and paid specifically to make these observations. Their job also included recording their observations in the diaries and preparing astronomical tables and yearly almanacs. A diary usually covered about six months of observation. The entries for each month typically include information on the following: the length of the previous month; lunar and solar eclipses; lunar and planetary conjunctions with each other or with Normal stars; solstices and equinoxes; heliacal risings and settings of planets and Sirius; meteors; and comets. In the diaries, the Babylonians also systematically recorded the six time-intervals termed by A. Sachs "Lunar Sixes". These may be described as follows. On the first day of the month the Babylonians recorded the time between sunset and moonset (*na*). Around the middle of the month they recorded four intervals related to the full moon: the time interval between moonset and sunrise when the moon set for the last time before sunrise (*ŠÚ*); the interval between sunrise and moonset when the moon set for the first time after sunrise (*na*); the interval between moonrise and sunset when the moon rose for the last time before sunset (*ME*); and the interval between sunset and moonrise when the moon rose for the first time after sunset (GE_6). Finally, near the end of the month the Babylonians recorded the time between moonrise and sunrise when the waning crescent moon was visible for the last time (*KUR*). In addition to the astronomical data, the diaries also contain some non-astronomical information: on the weather, the prices of six basic commodities, the height of the river Euphrates, and certain historical events.

It should be emphasized that although the major bulk of celestial phenomena referred to in the diaries are actual observations, some of the recorded events are not observations but rather predictions based on certain mathematical calculations. Sometimes this is clearly stated whereas on other occasions it is implicit, as in the case when the sky is mentioned as having been overcast.

Most of the available tablets containing the diaries are damaged to varying degrees

— often extensively. In some cases the date of the tablet is broken away. Such tablets can often be dated by using a unique combination of astronomical data that they record — for example, eclipses and lunar and planetary positions. This is how Sachs and Hunger determined many of the dates of the diaries, which they recently published in transliteration and translation in three volumes.[9] These volumes, which form the exclusive source for the Babylonian data of the current study, cover diaries from –651 (652 B.C.) to –60 (61 B.C.).

The following is an example of the diary reports, for the first seven days of the lunar month whose first day corresponds to B.C. 163 August 11 (parentheses denote editorial comment, square brackets indicate damaged text that has been restored by the editors, while the number at the beginning of each paragraph indicates the line number in the text):

> 1 Year 149 (Seleucid), king Antiochus. Month V, (the 1st of which was identical with) the 30th (of the preceding month), sunset to moonset: 10°, it was very low; measured (despite) mist.
>
> 2 Night of the 2nd, the moon was 1 cubit behind γ Virginis. Night of the 3rd, the moon was 1 cubit above α Virginis, the moon having passed 0.5 cubit
>
> 3 to the east. The 3rd, the north wind blew. Night of the 4th, the moon was 4 cubits in front of α Librae. The 4th, the north wind blew. Night of the 5th,
>
> 4 beginning of the night, the moon was 2.5 cubits below β Librae. The 5th, the east wind blew. Night of the 6th, beginning of the night, the moon was 20 fingers above β
>
> 5 Scorpii. The 6th, ZI IR (unidentified), the east wind blew. Night of the 7th, beginning of the night, the moon was 3 cubits in front of θ Ophiuchi,
>
> 6 the moon being 2.5 cubits high to the north, it stood 1 cubit 8 fingers in front of Mars to the west, the moon being 2 cubits high to [the north;]
>
> 7 last part of the night, Venus was 4 cubits below ε Leonis. The 7th, clouds were in the sky, ZI IR, the east wind blew.[10]

As seen in the above example, a typical diary starts with a mention of the Babylonian year and month. This is followed by a phrase stating that the first day of that month was either "identical with" or "followed"[11] the 30th of the preceding month, so indicating whether the previous month contained 29 or 30 days, the only lengths permitted by the Babylonian time-reckoning. After that there is a mention of the measured or predicted *na* interval, which is the time between sunset and moonset of the first day of the month — usually known as 'moonset lagtime' in modern terminology. This is one of the six quantities termed Lunar Sixes already mentioned.

During each month, the Babylonian observers recorded when the moon and planets passed near to each other or near to normal stars. In a diary, the relative position of a celestial body to another may be described by one of the terms "above" (*e*), "below" (*šap*), "in front of" (*ina IGI*), or "behind" (*ar*). The terms "behind" and "in front of" are roughly synonymous with "to the east of" and "to the west of", respectively, following the apparent rotation of the celestial sphere.

For the measurement of angles, such as the position of celestial bodies and magnitudes of eclipses,[12] the Babylonians used the units 'finger' (*SI*) and 'cubit' (*KÙŠ*), which contained twenty-four fingers in the Neo-Babylonian period.[13] It was previously suggested that the cubit was approximately equivalent to 2°.[14] However, a recent investigation of Babylonian measurements of close planetary conjunctions has shown that the cubit closely equalled 2.2°.[15] This last study has also shown that the Babylonians did not use horizon coordinates (altitude and azimuth), but there was little evidence to determine whether ecliptical or equatorial coordinates were used. However, because of the Babylonians' introduction of the concept of the zodiac around 400 B.C. it appears more reasonable to suppose that the Babylonian astronomers used an ecliptical system.

For the measurement of time intervals shorter than a day, such as the durations of the phases of an eclipse,[16] the Babylonians used the unit *uš*. According to Neugebauer, "The 'degree' (*uš*) is the fundamental unit for the measurement not only of arcs, especially for the longitude, but also for the measurement of time, corresponding to our modern use of right ascension. Therefore, 1 degree = 4 minutes of time".[17] Accordingly, Sachs and Hunger, who translate *uš* as "time degree", have converted all measurements in *uš* in the diaries, especially those of the Lunar Sixes, into time-degrees. We have confirmed, through the investigation of Babylonian records of lunar eclipse durations, that the modern equivalence of the *uš* is accurately 4 minutes and have shown that the definition of this unit showed no variations over the centuries covered by the Late Babylonian astronomical texts.[18]

4. *Determination of the Julian Date of First Visibility of the Lunar Crescent*

We have thoroughly scanned Sachs and Hunger's three volumes[19] and compiled a list of dates of Julian years and Babylonian months in which the moon was first sighted. This is not simply a list of each year and month cited in the extant diaries because, as already mentioned, the Babylonians did not depend solely on observation when determining the first day of the month, though this seems to have been the practice in ideal weather. The Babylonian astronomers did use mathematical methods for determining the first day of the month, at least when visibility of the lunar crescent was prevented by unfavourable weather conditions. Since our purpose was to collect dates of actual observations rather than predictions of first visibility of lunar crescents we have selected only the entries that contain explicit statements confirming that the moon was indeed sighted. Terms and phrases used by the Babylonians to indicate actual sighting of the moon include "visible", "seen", "first appearance", and "earthshine". Descriptions of the position of the moon or its brightness, such as "low", "could be seen", "was low to the sun", "faint" and "bright", are also indications of actual observations. Below are examples from different years of reported first sightings of the lunar crescent:

> Month V, (the 1st of which was identical with) the 30th (of the preceding month), first appearance of the moon; sunset to moonset: 12°; the moon was 2 cubits in front of Mercury.[20] [Julian date: B.C. 373 July 23]

> [Month V,] the 1st (of which followed the 30th of the preceding month), sunset to moonset: 15.5°; the moon was 1.66 cubits in front of α Virginis.[21] [Julian date: B.C. 334 August 12]

> Month IX, the 1st (of which followed the 30th of the preceding month), sunset to moonset: 15°, measured; the moon stood 1.5 cubits in front of Mercury to the west.[22] [Julian date: B.C. 274 December 4]

> Month IX, (the 1st of which was identical with) the 30th (of the preceding month), sunset to moonset: 17.5°; it was bright, earthshine, measured; it was low to the sun.[23] [Julian date: B.C. 204 December 10]

> [Month V, (the 1st of which was identical with) the 30th (of the preceding month), sunset to] moonset: [*nn*°]; it was faint, it was low to the sun; (the moon) [stood] 3 cubits in front of Mars, 5 cubits in front of Saturn to the west.[24] [Julian date: B.C. 171 August 9]

In order to confine ourselves to actual sightings of the lunar crescent, we have excluded all entries where the text contained explicit statements and terms implying invisibility of the moon, such as "I did not watch", "I did not see the moon", "overcast", "mist", and "clouds". We have also ruled out all entries in which the moonset lagtime or interval between sunset and moonset (*na*) is said to have been predicted as this might well be due to the fact that the moon was not seen. As an essential measure of extra caution, we have discounted any entry that does not contain a specific statement that the moon was seen, even if it does not contain any explicit or implicit indication to the contrary. Accordingly, the final list of acceptable entries, though numbering as many as 209 in total, was unavoidably only a small part of the original material. The following are examples of the kinds of entries that have been discarded for one or more of the reasons mentioned above:

> [Month XI, (the 1st of which was identical with) the 30th (of the preceding month),] sunset to moonset: 14°; there were dense clouds, so that I did not see the moon.[25] [Julian date: B.C. 453 February 12]

> Month VIII, the 1st (of which followed the 30th of the preceding month), sunset to moonset: 18.5°. Night of the 1st, clouds crossed the sky.[26] [Julian date: B.C. 271 November 2]

> Month II, (the 1st of which was identical with) the 30th (of the preceding month, sunset to moonset): 13°; dense clouds, I did not watch. Night of the 1st, [clouds] crossed the sky.[27] [Julian date: B.C. 256 April 23]

[Diaries from month VII to the end] of month XII, year 113, which is the year 177, King Arsaces. Month VII, the 1st (of which followed the 30th of the preceding month), sunset to moonset: 11.5°; mist [...].[28] [Julian date: B.C. 135 September 30]

Having collected all reliable dates of first sightings of the moon after conjunction, we made a preliminary conversion of all dates, which are given by Sachs and Hunger in terms of Julian year and Babylonian lunar month, to their full Julian equivalents. This could have been achieved using the specially prepared tables of Parker and Dubberstein[29] which cover the period 626 B.C. to A.D. 75. However, the use of these manual tables would not be very practical when a large number of data are involved. Therefore, we used only the intercalary scheme from these tables, i.e., the recorded positions of the additional months (which always followed the 6th or 12th month). We then integrated this scheme in a specially designed program that reads in the Babylonian date and converts it to its Julian equivalent, totally independently of the tables.

The program uses the lunar visibility criterion suggested by Schoch[30] to determine the expected dates of first visibility of the crescents. This is the criterion on which the tables of Parker and Dubberstein are based. The use of a specific lunar visibility criterion for this purpose is of no critical importance, because the converted dates, whether found manually by tables or by the program, could be considered only a first approximation anyway. The reason is that the date of actual observation of the crescent in any given month, which is the date that really matters for the purpose of this study, is not necessarily the same as that predicted by any theoretical calculation. For instance, a crescent that in theory should have been easily noticed could have set unseen because of unfavourable weather and its actual first visibility could have occurred the next evening. Therefore, in each instance the calculated date of first visibility must be checked against real observational data — usually in the form of a time or positional measurement from the month under consideration (see next section). In this way, one can be sure whether the theoretically calculated date is exact or in need of amendment. In practice, such amendments never exceeded a single day, but even such a seemingly small discrepancy is crucial for the purpose of crescent visibility studies.

In 136 of the 209 entries that we compiled, the measured moonset lagtime is given; since the lagtime changes from one day to another by an average of 54 minutes (some 13.5°), this quantity could be used to determine the exact date of first sighting of the lunar crescent. The following are two different explanatory examples:

Month III, (the 1st of which was identical with) the 30th (of the preceding month), the moon became visible behind Cancer; it (i.e. the crescent) was thick; sunset to moonset: 20°.[31]

This observation is from year –567. According to the date conversion program, the

Julian date of this event is –567 June 20. From our further computations, the moonset lagtime on that day was 89 minutes, i.e. 22.25 time-degrees, which is close to that given in the Babylonian text; hence B.C. 568 June 20 is confirmed to be the exact Julian date of observation.

> Month VIII, the 1st (of which followed the 30th of the preceding month), sunset to moonset: 17°; it could be seen while the sun stood there.[32]

This entry belongs to year –283. The calculated Julian date of this event is B.C. 284 October 26. However, the computed moonset lagtime on that date is 31 minutes, i.e. 7.75° time-degrees, which indicates that the exact date of observation was in fact the next day, i.e. B.C. 284 October 27; on this latter date the lagtime was 69 minutes, i.e. 17.25° time-degrees — almost the same quantity as measured by the Babylonians.

We found 9 entries where the difference between the measured and the computed lagtime was more than 4°, i.e. more than 16 minutes of time. The difference could well be due to inaccurate measurement of the lagtime or scribal error in the original text, and does not necessarily indicate an error in the date. Measurement of the *na* interval would be a difficult task since the young crescent moon can be seen only for a short time about midway between sunset and moonset. However, as a measure of caution, we re-checked these entries using additional data from the text. For this purpose, we used observations of lunar horizontal separation (i.e. when the moon is "behind", "east", "in front of", or "west") from a star or planet recorded during the same lunar month. Because the moon traverses about 13° every day, the date of any lunar conjunction in the month can be exactly determined, and this date can be used as a reference for verifying the date of the first day of the month, i.e. the date of the observation. However, if the text did not mention the horizontal separation we used the vertical separation (i.e. when the moon is "above" or "below") because the latter would be given only when the moon was horizontally close to the planet or star.

In the other 73 of the compiled 209 entries, the observed lagtime was missing, mostly because the text is broken away. In this case, we used other astronomical data from the same month to verify the date, exactly as in the case of the 9 entries mentioned above. Table 1 includes the exact Julian dates of the 209 Babylonian observations of the lunar crescent mentioned in the astronomical diaries.

For calculating the lunar coordinates, we designed a program that uses the semi-analytical lunar ephemeris ELP2000-85.[33] Although Chapront-Touzé and Chapront[34] suggest that ELP2000-85 is valid over a time span of several thousand years, using this theory for ancient times requires a significant modification. The ELP2000-85 solution assumes a value of –23.895″ /cy^2 for the tidal secular acceleration of the moon. However, recent results from lunar laser ranging (LLR) suggest a higher lunar acceleration of –25.88 ± 0.5″ /cy^2.[35] In a recent communication to one of the present authors,[36] J. L. Williams of the LLR team claims that consistent results for lunar acceleration are being found in the range –25.8 to –26.0″ /cy^2. Although the difference between these recent results and that assumed by ELP2000-85 may seem

TABLE 1. The 209 Babylonian observations of first visibility of the lunar crescent.

No.	Year	Month	Day	No.	Year	Month	Day	No.	Year	Month	Day	No.	Year	Month	Day
1	−567	4	22	54	−284	11	6	107	−192	3	18	160	−143	8	11
2	−567	5	22	55	−283	10	27	108	−192	9	11	161	−143	9	9
3	−567	6	20	56	−281	11	4	109	−190	3	26	162	−143	10	9
4	−566	2	12	57	−277	3	28	110	−190	5	24	163	−142	11	26
5	−566	3	14	58	−277	4	26	111	−189	10	9	164	−141	5	23
6	−418	10	20	59	−277	5	26	112	−189	11	7	165	−141	10	17
7	−381	5	6	60	−273	12	4	113	−188	4	2	166	−140	4	12
8	−381	7	4	61	−266	10	19	114	−187	10	16	167	−140	7	9
9	−378	10	27	62	−266	11	18	115	−187	11	14	168	−140	12	3
10	−374	1	20	63	−264	9	26	116	−185	3	2	169	−139	2	1
11	−374	3	20	64	−255	3	25	117	−183	5	7	170	−137	12	31
12	−372	2	27	65	−255	9	17	118	−183	8	4	171	−136	9	22
13	−372	3	28	66	−251	10	3	119	−183	10	31	172	−134	10	30
14	−372	7	23	67	−250	2	28	120	−181	2	15	173	−133	2	25
15	−370	8	1	68	−249	8	13	121	−179	3	24	174	−133	8	20
16	−370	10	28	69	−246	1	15	122	−179	7	20	175	−133	9	19
17	−368	7	10	70	−246	4	14	123	−178	8	8	176	−133	10	19
18	−366	6	18	71	−246	5	14	124	−178	9	6	177	−132	3	15
19	−366	8	17	72	−246	10	8	125	−178	10	6	178	−132	10	7
20	−346	12	2	73	−245	5	3	126	−176	9	13	179	−131	10	26
21	−345	3	1	74	−245	7	1	127	−176	10	13	180	−129	7	9
22	−342	12	17	75	−237	7	3	128	−175	5	9	181	−124	12	7
23	−333	6	14	76	−237	8	1	129	−175	12	1	182	−123	2	4
24	−333	8	13	77	−234	9	26	130	−173	11	10	183	−123	6	2
25	−332	9	29	78	−234	11	24	131	−173	12	9	184	−119	4	19
26	−328	10	13	79	−233	2	20	132	−172	2	6	185	−119	6	17
27	−328	12	12	80	−233	3	21	133	−170	8	9	186	−118	5	8
28	−324	4	6	81	−232	10	3	134	−170	10	8	187	−117	10	22
29	−324	7	3	82	−231	2	28	135	−170	11	7	188	−111	3	22
30	−324	8	2	83	−225	1	23	136	−169	2	3	189	−111	6	19
31	−324	9	30	84	−218	10	28	137	−169	5	2	190	−111	8	18
32	−322	12	7	85	−217	2	23	138	−168	8	17	191	−107	4	7
33	−321	1	5	86	−210	7	4	139	−168	12	13	192	−105	4	16
34	−321	2	3	87	−209	5	24	140	−164	6	5	193	−105	5	15
35	−321	4	3	88	−207	4	2	141	−164	10	31	194	−105	6	13
36	−321	7	30	89	−207	5	1	142	−163	4	25	195	−105	9	9
37	−321	8	29	90	−203	12	10	143	−163	5	26	196	−105	10	9
38	−307	6	26	91	−201	12	18	144	−163	11	19	197	−104	8	29
39	−307	8	24	92	−200	3	16	145	−162	3	16	198	−96	5	5
40	−302	7	29	93	−198	6	21	146	−162	8	11	199	−95	5	24
41	−302	8	28	94	−197	3	14	147	−162	9	10	200	−87	7	23
42	−302	10	27	95	−197	10	7	148	−161	9	29	201	−87	9	20
43	−302	11	26	96	−197	11	5	149	−158	6	29	202	−86	3	17
44	−302	12	26	97	−196	2	2	150	−158	8	26	203	−86	11	7
45	−301	1	24	98	−195	11	12	151	−156	12	1	204	−83	7	9
46	−301	6	19	99	−194	1	11	152	−154	1	18	205	−77	6	4
47	−294	5	4	100	−194	6	7	153	−151	3	15	206	−77	8	2
48	−293	1	25	101	−194	10	3	154	−149	11	14	207	−77	9	1
49	−291	5	1	102	−193	4	28	155	−145	1	9	208	−77	10	30
50	−291	6	29	103	−193	5	28	156	−145	2	7	209	−73	7	19
51	−291	8	26	104	−193	10	22	157	−144	9	21				
52	−289	6	8	105	−192	1	19	158	−144	10	20				
53	−286	6	4	106	−192	2	17	159	−144	11	18				

small, it does nevertheless accumulate significant errors over a long period as in the case of the Babylonian data. We have remedied this situation by using a special formula given in the *Astronomical almanac* which accounts for the deficiency in the tidal secular acceleration of the moon by modifying the Julian day number of the event so that the computed lunar coordinates are for a lunar acceleration of $-26''/\text{cy}^2$.[37]

In order for the calculations to be valid for an ancient epoch such as the Babylonian, it is also necessary to make allowance for the cumulative effect of changes in the length of the day (ΔT) which results from variations in the earth's rate of rotation due to tides and other causes.[38] For example, ΔT is estimated to have been as much as about 16800 seconds (4.66 hours) in the year −500 which corresponds to changes of about 2.5° and 0.2° in the lunar longitude and latitude, respectively. We have incorporated into our calculations ΔT using the values recently derived by Stephenson and Morrison[39] from their analysis of historical records of astronomical events — mainly eclipses, including those from Babylon.

We computed the solar coordinates using the solution VSOP82 (stands for *Variations Séculaires des Orbites Planétaires*)[40] and the planetary positions using the analytical theory VSOP87.[41] We designed a special program for calculating the stellar coordinates.

5. *The Babylonian Criterion of First Visibility of the Lunar Crescent*

The accurate prediction of the evening of first visibility of the new crescent was of major significance for the Babylonians. Indeed, this matter was of such importance that it was the main goal of the Babylonian lunar theory in the Seleucid period (311 – 64 B.C.).[42] The Babylonians succeeded in formulating a truly mathematical lunar theory which they used for predicting various parameters of the lunar motion, as found recorded in the lunar ephemerides they prepared.

Modern investigators of the problem of first visibility of the new crescent, who are not themselves scholars of Babylonian astronomy, have systematically claimed that the Babylonian conditions of visibility were that the age of the new moon is more than 24 hours and that the arc of separation(s) should be equal to or greater than 12°, i.e. that the moon sets at least 48 minutes after sunset. This supposed Babylonian criterion is also often cited as being only $S \geq 12°$. It seems that Bruin[43] was the first modern researcher to attribute this criterion to the Babylonians and that all subsequent researchers who reiterated this claim were simply relying on his account.[44] However, it should be noted that Bruin cited no reference in support of his claim. Bruin seems to have suggested it because he noted that the simple rule of $S \geq 12°$ was used by Arab astronomers from the seventh century onward; he believed that it might have transmitted to them from the Hindus who would have learned it from the Babylonians. However, Bruin's claim with regard to the Babylonian condition of lunar visibility is, at best, inaccurate. The 12° equatorial difference is indeed the crescent visibility criterion adopted by the Indian *Suryasiddhanta* (*c.* 600) and the *Khandakhadyaka* (650), as pointed out by King.[45]

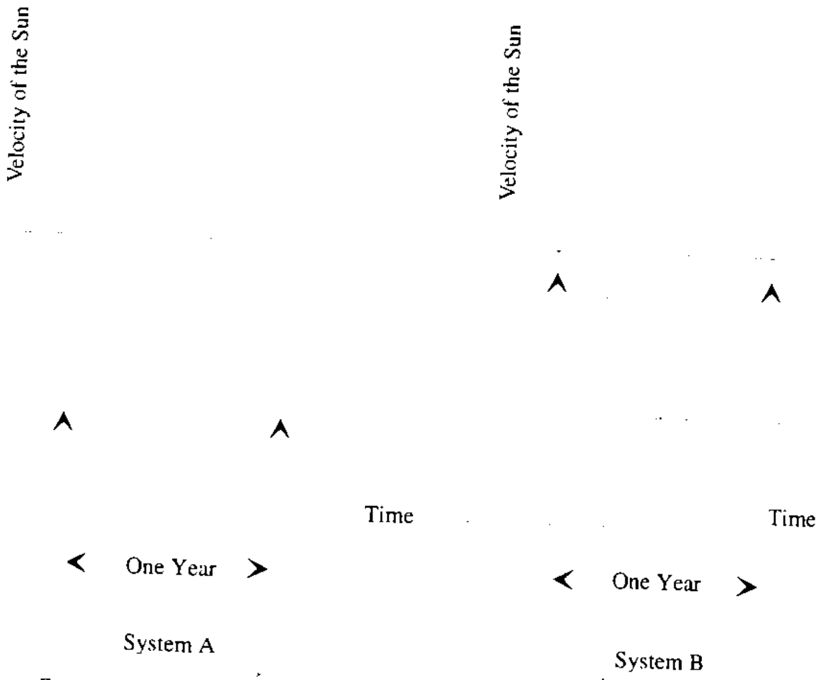

FIG. 1. The two representations of solar motion in the Babylonian lunar theory.

However, even though Babylonian astronomical knowledge had passed to the Indians (by way of the Greeks), this does not necessarily imply that this was the Babylonian criterion of crescent visibility.

Study of the Babylonian lunar ephemerides has revealed that they are based on two somewhat different versions of lunar theory, usually referred to as "System A" and "System B". According to System A, the sun moves with constant velocities on two different arcs of the ecliptic, whereas System B assumes that the solar velocity changes with time in a linear zigzag function.[46] The difference between the two theories is usually represented by Figure 1.

It is interesting to note that although System B must have been an improvement of System A, both Systems were used simultaneously throughout the period 250–50 B.C. in preparing ephemerides. Neugebauer notes that such a practice, which is contrary to our modern scientific concepts where new theories replace old ones, is yet more prominent in the planetary theory.[47] The lunar ephemerides were used by the Babylonians to predict the first and last visibility of the moon. A comparative list of the main columns of computations of a complete ephemeris in the two systems is given in Table 2.[48]

Although the existence of procedure texts that give criteria for determining the first and last visibility of the moon is hard to doubt, so far, unfortunately, no such texts have come to light. Therefore, it is only through the analysis of individual

TABLE 2. The columns of astronomical calculations included by Babylonian astronomers in each ephemeris of System A and System B. As seen, some parameters are calculated in ephemerides of both Systems whereas others are restricted to one System or the other. Although the last four quantities are missing from the tables of System A, preserved procedural texts tell us they were calculated: they would be necessary for finding the lagtime.

System A	System B
Dates	
Relative velocity of the moon with respect to the sun(?)	
	Velocity of the sun
Longitude of the moon	
Length of daylight	
	Half length of the night
Latitude of the moon	
Magnitude of eclipses	
Velocity of the moon	
Length of the month in first approximation	
	Correction related to the next column
Correction in the length of the month caused by the variability of solar velocity	
Second correction to the length of the month	
Length of the month	
	Date of syzygy, midnight epoch
Date of syzygy, evening epoch	Date of syzygy, evening or morning epoch
	Time difference between syzygy and sunset or sunrise
	Elongation of first or last visibility
	Influence of the obliquity of the ecliptic
	Influence of the latitude
Duration of first or last visibility (lagtime)	

cases in the ephemerides that certain criteria can be concluded.

Contrary to what is commonly assumed, Neugebauer[19] found from the study of extant ephemerides that the moonset lagtime alone could not have been used as the visibility criterion by the Babylonians in any of the two Systems. He suggests that a criterion of the following form might have been used by the Babylonians for both Systems:

$$\text{elongation (L)} + \text{moonset lagtime (in degrees) (S)} > \text{constant}.$$

Neugebauer suggests that the rationale behind the inclusion of the elongation in the criterion of first visibility would be that the elongation measures, in addition to the angular distance between the sun and moon, the width of the visible crescent. Therefore, this criterion would imply that the chance of sighting the new crescent increases with the width of the crescent and with the time for which the crescent remains above the horizon before setting.

As for the value of the constant in the above criterion, Neugebauer found from his study of preserved texts that in the case of System A the constant could have been about 21°. In other words, the Babylonian visibility criterion for System A is:

$$L + S > 21°.$$

In the case of System B, Neugebauer found two ephemerides that suggest a value of about 23° for the constant, whereas another suggests ≥ 20° and a fourth accepts a value as low as ≥ 17°. This represents a considerable range.

Interestingly, Neugebauer notes that the moonset lagtime might have been used alone for predicting the visibility of the new moon in some cases. He concludes this from the existence of isolated lists of lagtimes that seem to have been collected for several years in succession. The lowest values found in these texts are 11.33°, 11.66°, and 11.83°, and these are followed by a phrase of unknown meaning. The highest value of lagtime given is 25.16° without alternative, although an ephemeris preserved for the same year accepts instead 12°. One alternative solution of 20.5° for a full month (30 days) and 10.5° for a hollow month (29 days) is also given.[50]

We have found that the smallest value of L + S in the 209 observations is 22° (observation 89), which is very close to the limits of 21° and 23° suggested by Neugebauer for Systems A and B respectively. The highest value of L + S that we have found is 57.9° (observation 143). Therefore, while exceeding the 22° limit does not ensure visibility of the lunar crescent, this value may possibly have been used by the Babylonians as the lowest limit for the visibility of the crescent.

The latitude of Babylon is about 32.6° N. To test the reliability of the above criterion that the Babylonians might have used, we applied it to the observations of Table 1 as well as all entries of latitudes within the range ± (30° – 35°) from the modern compilations.[51] We assumed that the Babylonian criterion was L + S ≥ 22°, as this is the smallest value in the Babylonian data. We found that the quantity L + S is less than 22° for only 2 of the 231 positive observations of latitudes close to that of Babylon. But while this criterion misjudges only 0.9% of the *positive* observations, it has 7 of the 19 *negative* observations in the visibility zone, i.e. L + S greater than 22°. The latter result represents a very high percentage of error, 36.8%. The unreliability of this criterion becomes even more manifest when applied to the data from all latitudes. Five of the total of 399 Babylonian and modern positive observations, i.e. 1.3%, are wrongly placed according to the Babylonian criterion, but as many as 37 of the 81 negative observations, i.e. 45.7%, contradict the visibility condition. Certainly, this would be a very bad global criterion.

There have been modern attempts to formulate modern crescent visibility criteria that would predict the dates when the crescent could have been visible in Babylon. These attempts were originally triggered by interest in determining the beginnings of the Babylonian months, which would help in establishing the equivalent dates of Babylonian records. One such solution was first attempted by Karl Schoch, who designed tables for determining the evening of the first sighting of the lunar crescent that are applicable to all places whose latitudes differ little from that of Babylon. Schoch also presented his lunar visibility tables, following Fotheringham,[52] in the form of a curve of true lunar altitude (h) (parallax is not accounted for) versus the azimuthal difference between the sun and moon (ΔZ) at sunset, so that the new moon would be first visible on the first evening after conjunction in which the moon falls above the curve (see Table 3).[53] However, the criterion of Schoch suffers from the important flaw of being based on both observations and predictions of the lunar crescent.[54] Even Schoch's identification of what he considered to have been observations was not totally sound. For instance, Schoch states that "The most valuable observations for my purpose are the most ancient, belonging to a time when the Babylonians were unable to compute the appearance of the crescent, i.e. the time from Rim-Sin to Ammizaduga and from Nebuchadnezzar to Xerxes".[55] But the fact that the Babylonians were at some stage of their history unable to predict the first appearance of the crescent does not necessarily mean that they did not follow some simple rules in fixing their calendar, the most probable and simplest of such rules being that the month would be of either 29 or 30 days. If so basic a rule was followed, then the lengths of the Babylonian months determined according to this rule would have no implications whatsoever for the visibility of the moon. (It should be stressed that the skies of Babylon are often cloudy in winter, for example.) It was exactly to avoid using such pseudo-observational data that for the present project we collected only actual observations of the lunar crescent. Although Fotheringham[56] expresses his confidence in Schoch's criterion for computing the first visibility of the lunar crescent at Babylon, Schoch's use of predictions in addition to observations in setting his criterion has been criticized by O. Neugebauer.[57] It seem fair to conclude that Schoch's solution can be regarded as neither observational nor theoretical, and hence it is likely to lead to errors in predicting the dates of first sightings of the lunar crescent in Babylon.

Another criterion for determining the first visibility of the lunar crescent at Babylon was suggested by P. V. Neugebauer. This solution uses the same two parameters employed by Schoch, i.e. ΔZ and h, but the suggested curve lies a little below that of Schoch for smaller ΔZ and slightly above it for larger ΔZ.[58] However, the differences between both curves are too small to be of any significance in practical use. Neugebauer's curve also extends to 23° of ΔZ, in contrast to that of Schoch which covers only up to 19° of ΔZ (see Table 3 for both criteria). We did not come across any other modern criterion that is based on Babylonian data or is designed to predict the lunar visibility in Babylon in particular. Researchers into the Babylonian

TABLE 3. The criteria of K. Schoch and of P. V. Neugebauer. At any specified azimuthal difference from the sun, the crescent is expected to be visible when the moon is not lower than a critical true altitude at sunset.

True azimuthal difference (ΔZ)	Minimum true lunar altitude (h) Schoch	Neugebauer
°	°	°
0	10.7	10.4
1	10.7	10.4
2	10.6	10.3
3	10.5	10.2
4	10.4	10.1
5	10.3	10.0
6	10.1	9.8
7	10.0	9.7
8	9.8	9.5
9	9.6	9.4
10	9.4	9.3
11	9.1	9.1
12	8.8	8.9
13	8.4	8.6
14	8.0	8.3
15	7.6	8.0
16	7.3	7.7
17	7.0	7.4
18	6.7	7.0
19	6.3	6.6
20	-	6.2
21	-	5.7
22	-	5.2
23	-	4.8

calendar have relied on one or the other of the above criteria (for example, Parker and Dubberstein[59] used Schoch's model while Huber[60] opted for that of Neugebauer).

We have examined both criteria of Schoch and Neugebauer using the 209 observations that we have collected from the Babylonian diaries. Because these are real observations, they can serve as a very reliable indicator of the accuracy of both criteria. We have plotted in Figure 2 the visibility curves of Schoch and Neugebauer as well as the 209 Babylonian observations. The graph shows that both models are reasonably good in predicting the observations. Of the 209 positive observations, only 8 fell below the visibility curves. In other words, according to the criteria of Schoch and Neugebauer about 3.8% of the sighted crescents would have been invisible. However, if the visibility curve is drawn downwards starting from about $h = 9.45°$ for $\Delta Z = 0°$, then all of the observations would be above the visibility curve, i.e. in the visibility zone.

It should be stressed, however, that the fact that this modified curve would have almost all positive observations in the visibility zone does not tell us anything about the suitability of this criterion for hypothetical negative observations from Babylon. In other words, it is obvious that while lowering the dividing line would include all

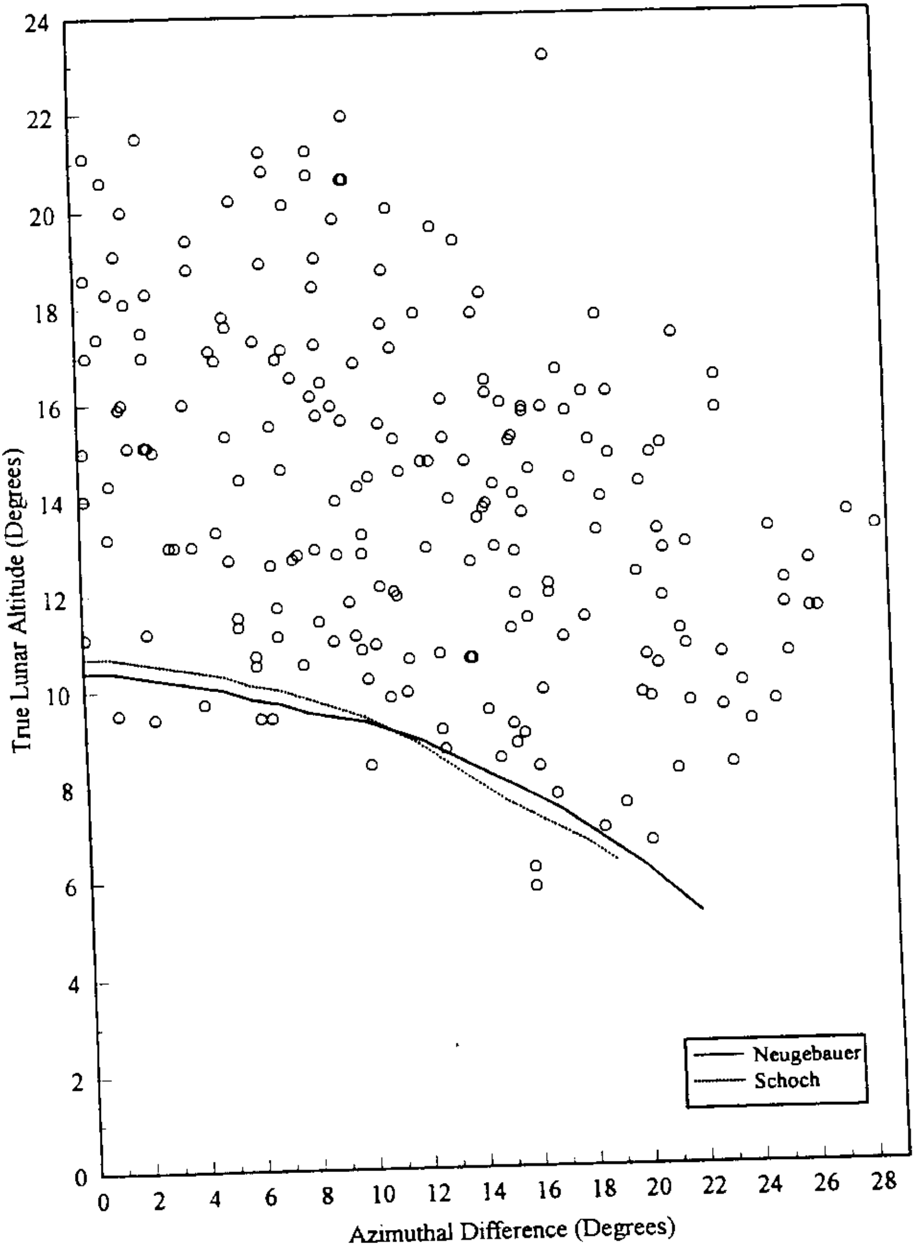

FIG. 2. The visibility criteria of Schoch and P. V. Neugebauer together with the Babylonian positive observations.

the positive observation in the visibility zone, i.e. above the curve, the curve will certainly become so low that it would have more negative observations in its visibility zone than would the original curves. While lowering the criterion curve would definitely give better results as far as positive observations are concerned, it would also increase the number of Babylonian months that actually began one day later than the solution predicts. This drawback in the criteria of Neugebauer and Schoch would have become manifest if the Babylonian data included actual negative observations in addition to the positive.

A realistic evaluation of the h – ΔZ criteria can be made with the help of the modern lists of observations which, unlike the Babylonian collection, include negative in addition to the positive observations. Neugebauer's curve misjudges 22 of the 81 negative observations, i.e. 27.2%, and misses 50 of the 399 positive, i.e. 12.5%. Obviously, this criterion cannot be considered satisfactory. Schoch's curve would not give a significantly different results.

While Schoch suggested that his model is applicable to all latitudes close to that of Babylon, Fotheringham claimed that his h – ΔZ criterion is independent of the geographical latitude of the observer. We have used the data in Table 1 as well as the modern lists to investigate whether or not the h – ΔZ criterion depends on the observer's latitude. We have, therefore, separated the observations into two categories according to the geographical latitude of the observers, including in the first category only the observations made from latitudes ± (30° – 40°). These consisted of 372 observations, 310 positive and 62 negative. The second group included all the other 108 observations, 89 of which are positive and the remaining 19 negative.

We have plotted in Figure 3 the mid-latitudes data and Neugebauer's form of the h – ΔZ criterion. The curve has 27 of the 310 (7.7%) positive observations (denoted by circles) in the invisibility zone and 13 of the 62 (21%) negative observations (denoted by crosses) in the visibility zone. Figure 4 is similar to Figure 3 but it includes the observational data from all the latitudes other than ± (30° – 40°). Here the curve of Neugebauer has as many as 9 of the 19 (47.4%) negative observations and 26 of the 89 (29.2%) positive in the wrong zone (though Figures 3 and 4 may show smaller numbers of points because of coinciding data points).

It seems from Figures 3 and 4 that Neugebauer's criterion gives much larger errors when applied to latitudes away from that of Babylon. This shows that, contrary to Fotheringham's assertion, this type of solution is latitude-dependent. This and the high percentage of error that all forms of this criterion give cannot be overcome by simply lowering or raising the curve or even changing its shape. Any such changes can improve the reliability of the criterion with respect to some of the data but only at the cost of worsening its assessment of the rest. For instance, lowering the curve would decrease the number of positive observations that are already in the invisibility area, but this would then raise more negative observations to the visibility zone. Similarly, any change to make the criterion more suitable to a certain range of latitudes would make it more unreliable for other latitudes. The observational

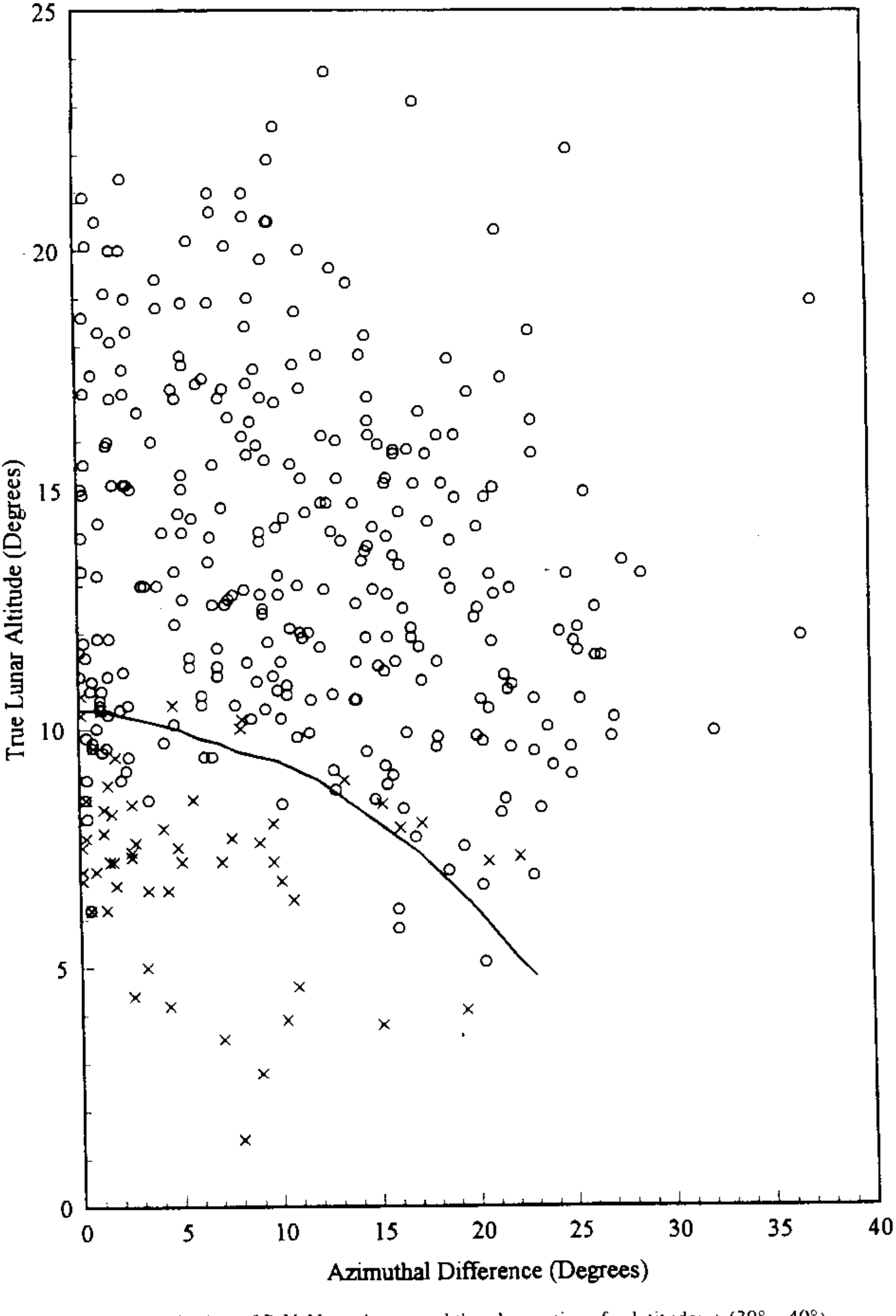

FIG. 3. The criterion of P. V. Neugebauer and the observations for latitudes ± (30° – 40°).

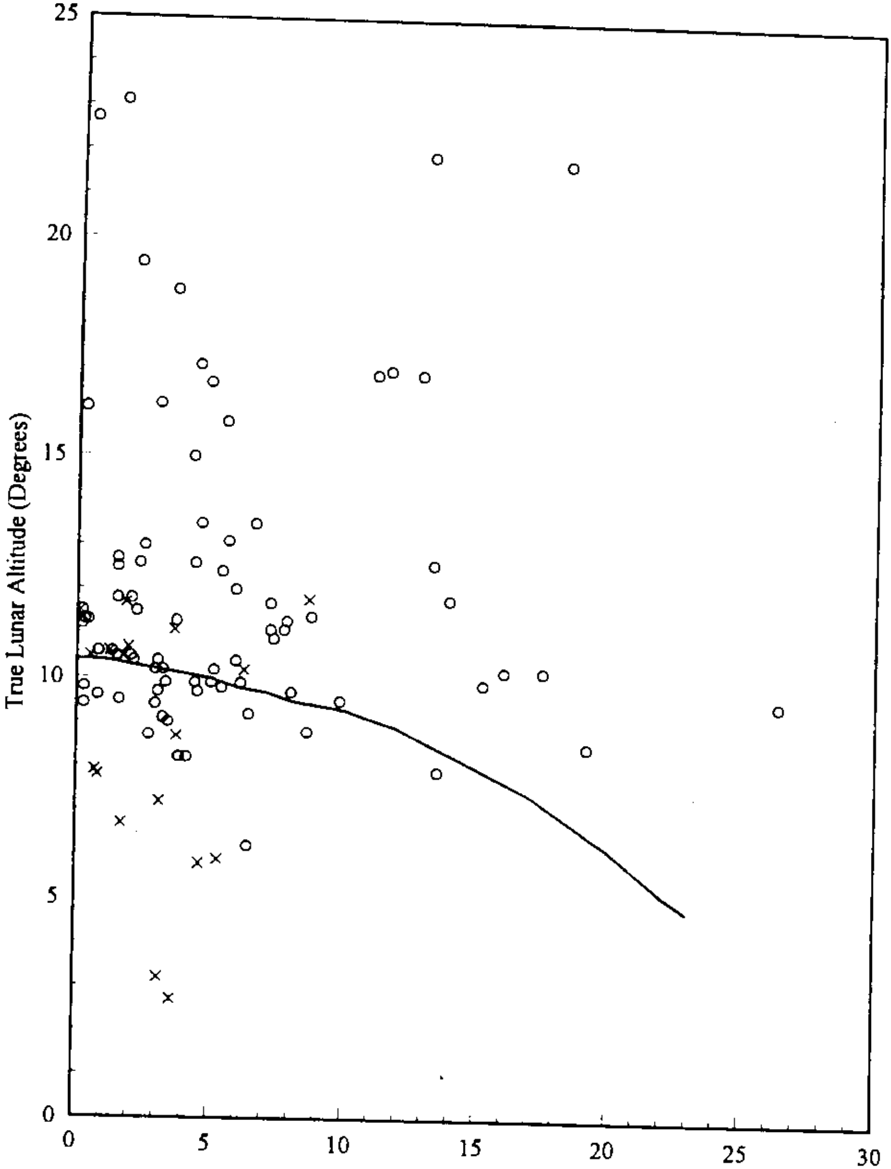

FIG. 4. The criterion of P. V. Neugebauer and the observations for latitudes other than ± (30° – 40°).

data show that at azimuthal difference of 0.5° a crescent that is as low as 6.2° has been seen. This is not an isolated observation. Another observer saw a crescent of 8.1° altitude and 0.4° azimuthal difference with the naked eye only. On the other hand, still for very small ΔZ, many crescents that are higher than 10° or even 11° have been missed. Therefore, it seems fair to say that the h – ΔZ criterion, in its present form, is itself inherently of limited utility for predicting the first visibility of the lunar crescent on the global level. Neither this criterion nor the L + S that *may* have been used by the Babylonians can be used confidently for predicting the first visibility of the lunar crescent.

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